PHYSICAL CAPITAL

How Large is the Contribution of Physical Capital Accumulation to Income Growth?

1. The Model
2. Calibration
3. Experiments
4. Growth Decomposition

THE MODEL

- Ultimate Goal: Understand why poor countries are poor.
- Goal here: What do the data suggest on the sources of growth?
- Consider a growth model that fits the data.

(Neo-classical) (US, 1960-2005)

("Neoclassical" is just a name, like "Marxist" or "Keynesian".)

- $Y_t = F(\text{inputs})$: We want to specify this.

t=year (2008, 2009, ...)

Y_t: Total output (units produced)

"inputs": labor (L_t), physical capital (K_t), ...

THE MODEL

- Examples: $Y_t = L_t$.

  $Y_t = AL_t$

  $Y_t = A(L_t)^{\alpha}$, $0 < \alpha < 1$.

  $Y_t = A_t(L_t)^{\alpha}$.

  $Y_t = A_t(K_t)^{1-\alpha}(L_t)^{\alpha}$. (Cobb-Douglas)

  $Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}$. (Cobb-Douglas)

  (If A_t becomes twice, Y_t also becomes twice.

  If L_t becomes twice, $Y_t \uparrow$ but becomes less than twice.

  If K_t becomes twice, $Y_t \uparrow$ but becomes less than twice.

  If both L_t and K_t become twice, Y_t also becomes twice.)

THE MODEL

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$

- Many economists find Cobb-Douglas reasonable. Why? Fits well to data.
- Y_t: output (measured by constant $), in data
- L_t:labor input (measured by manhours or #workers), in data
- K_t: physical capital stock, such as computers, machines, buildings, etc. (measured by constant $), not in data. Sometimes we do have in data, but it is constructed from the method that we discuss below.
- A_t: productivity (often called "total factor productivity"). Basically "everything else." Including education, R&D, efficiency of the market, …. Completely unobservable.

THE MODEL

$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}$. (Cobb-Douglas)

- We have to find out how Y_t, A_t, L_t, K_t grow.
- A_t and L_t grow exogenously. That is, our model does not explain why they grow.
  - $A_{t+1} = A_t(1 + g_A)$. (g_A: constant growth rate of A_t, such as 0.02.)
  - $L_{t+1} = L_t(1 + g_L)$. (g_L: constant growth rate of L_t, such as 0.01.)
  - For example, if $g_A = 0.02$,

A_{2006}	A_{2007}	A_{2008}	A_{2009}
100	**102**	**104.04**	**106.12**

THE MODEL

$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}$. (Cobb-Douglas)

- K_t grows following

$$K_{t+1} = I_t + (1-\delta)K_t.$$

  - I_t: investment (measured by constant \$), in data
    vs. C_t: consumption (measured by constant \$), in data
    $Y_t = C_t + I_t$.
  - δ: depreciation rate (5% or so)
  - For example, if $\delta = 0.05$,

	2006	2007	2008	2009
K_t	\$100	**\$115**	**\$129.25**	
I_t	\$20	\$20	…	

THE MODEL

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$

- $s_t = I_t/Y_t$: investment rate
- Then $I_t = s_t Y_t$, so $K_{t+1} = s_t Y_t + (1-\delta)K_t$.
- Done! Now we can link all variables so far.

THE MODEL

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$

- Assuming $\alpha = 1/3$, $g_A = 0.02$, $g_L = 0.01$ and $\delta = 0.05$,

	2006	2007	2008	2009
A_t	100	102	104.04	106.12
L_t	1,000	1,010	1,020.1	1,030.3
K_t	$100	?		
Y_t	**$46,416**			
s_t	0.2	0.2	0.2	0.2
I_t	**$9,283**			
C_t	**$37,133**			

- Our goal is to make this table for the U.S.

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CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- Issue: α, g_A, g_L and δ? How do we construct the "table"?
- We can't calibrate without further assumptions.
- **Assumption 1**: "Balanced Growth Path" (Each variable grows at a constant rate.)
- This is reasonable. (The growth rate doesn't rise or decline forever. Maybe it does in the short run, but eventually it will be stable.)
- **Assumption 2**: s_t is constant.
- This is also reasonable from the U.S. data.

CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- **Data**: $s_t = 0.20$. (Average of 1960-2005. For each year t, divide gross domestic investment, in NIPA Table 1.1.5, by GDP, in NIPA Table 1.1.)
- **Data**: $g_L = 1.1\%$. (Average of 1960-2005, NIPA Table 7.1.)
- **Data**: $g_Y = 3.3\%$. (Average of 1960-2005, NIPA Table 7.1. This is the sum of g_L and "GDP per capita in chained dollars".)
- **Empirical Studies**: $\delta = 0.05$. (Common assumption in macro. Curious? Please take my PhD macro course!)

CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- **Calibration of** K/Y **Ratio**
  - **Data**: $\delta K/Y = 0.12$ (average of 1960-2005, NIPA Table 1.1. This is a ratio of "consumption of fixed capital" to Y.)
  - Since we know $\delta = 0.05$, we have $K/Y = 2.42$.
  - Now we measured the unobservable K with this model!
  - On BGP, $g_Y = g_K$, i.e., Y_t and K_t grow at the same rate (since K/Y is constant).

CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- **Calibration of** α

- **Interest rate** = marginal product of physical capital
- Each unit of K is paid its marginal product. ($1 → $\$(1 + r)$)
- $r_t = \frac{\partial Y_t}{\partial K_t} = A_t \alpha (K_t)^{\alpha-1}(L_t)^{1-\alpha}$
- So total "K income" is $r_t K_t = A_t \alpha (K_t)^{\alpha}(L_t)^{1-\alpha} = \alpha Y_t$.
- Similarly, total "L income" is $(1 - \alpha)Y_t$.

CALIBRATION

- α: income share of physical capital (interests, stock returns, …)

  $1 - \alpha$: income share of labor (salaries, …)
- **Data**: (income share of K)=0.32 (NIPA Table 1.12. This is a fraction of "compensation of employees" out of "GDP" minus "proprietors' income" minus "taxes on production and imports".)
- **Result**: $\alpha = 0.32$
- We can also compute the interest rate. Since $r_t K_t = \alpha Y_t$, we have

  $r_t = \frac{\alpha Y_t}{K_t} = \frac{\alpha}{K/Y} = \frac{0.32}{2.42} = 0.132.$
- **Result**: $r = 0.132$

CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- **Calibration of** g_A

- Recall: Growth rate of X_t is (i) $\frac{X_{t+1}}{X_t} - 1$ or (ii) $\log(X_{t+1}) - \log(X_t)$.
- $\log(Y_t) = \log[A_t(K_t)^{\alpha}(L_t)^{1-\alpha}]$

  $= \log(A_t) + \log(K_t)^{\alpha} + \log(L_t)^{1-\alpha}$

  (since $\log(XY) = \log X + \log Y$)

  $= \log(A_t) + \alpha\log(K_t) + (1 - \alpha)\log(L_t)$ (since $\log(X^a) = a\log X$)
- $\log(Y_{t+1}) = \log(A_{t+1}) + \alpha\log(K_{t+1}) + (1 - \alpha)\log(L_{t+1})$

CALIBRATION

- $\log(Y_{t+1}) = \log(A_{t+1}) + \alpha\log(K_{t+1}) + (1 - \alpha)\log(L_{t+1})$

  $\log(Y_t) = \log(A_t) + \alpha\log(K_t) + (1 - \alpha)\log(L_t)$
- $\log(Y_{t+1}) - \log(Y_t) = \log(A_{t+1}) - \log(A_t)$

  $+\alpha[\log(K_{t+1}) - \log(K_t)] + (1 - \alpha)[\log(L_{t+1}) - \log(L_t)]$
- Recall $\log(Y_{t+1}) - \log(Y_t) \approx (Y_{t+1} - Y_t)/Y_t$.
- $g_Y = g_A + \alpha g_K + (1 - \alpha)g_L$ (Time subscript is omitted. It's BGP!)
- **Result**: $g_A = g_Y - \alpha g_K - (1 - \alpha)g_L$

  $= 0.033 - 0.32 \times 0.033 - (1 - 0.32) \times 0.011 = 0.015$

CALIBRATION

$$Y_t = A_t(K_t)^{\alpha}(L_t)^{1-\alpha}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = A_t(1 + g_A).$$
$$L_{t+1} = L_t(1 + g_L).$$
$$K_{t+1} = s_t Y_t + (1 - \delta)K_t.$$

- We quantified the model.

- $s_t = 0.20$
- $g_L = 1.1\%$
- $g_Y = g_K = 3.3\%$
- $g_A = 1.5\%$
- $\delta = 0.05.$
- $K/Y = 2.42$
- $\alpha = 0.32$

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S. Choi, EconS 427: Development, Fall 2009, WSU

EXPERIMENTS

$$Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = 1.015A_t.$$
$$L_{t+1} = 1.011L_t.$$
$$K_{t+1} = s_tY_t + (1-0.05)K_t.$$

- We also know on the BGP: $s_t = 0.20$, $g_Y = g_K = 3.3\%$, $K/Y = 2.42$.

- Plan:

1. Long-run trend
2. How to measure unobserved $\{A_t\}$ and $\{K_t\}$?
3. What will happen if the U.S. doubles s_t?
4. What will happen if there is a one-time capital inflow to the U.S.?

EXPERIMENTS

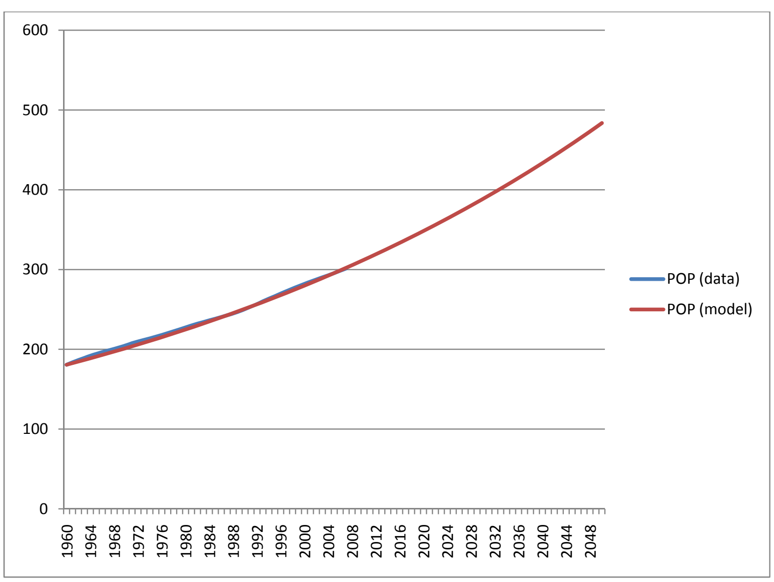
- Data Source: Penn World Table (http://pwt.econ.upenn.edu/php_site/pwt_index.php, latest version)
- Steps 1-2: Choose your country
- Step 3: Choose
  - POP: Population (thousands), L_t
  - ki: Investment share (%), s_t
  - rgdpl: Real GDP per capita (2005 constant prices), Y_t/L_t
- Steps 4-5: Choose years of your interests (1960-2005 in our example)
- We want to work with Excel. Choose CSV.
- As instructed at website, save the data part at Notepad as txt.

EXPERIMENTS

- Open Excel. Choose "Open" and select "All Files". Select your txt file.
- Choose "Delimited" and "Comma".
- Long-run trend of $\{L_t\}$
  - Obtain the "long-run trend" of $\{L_t\}$ by applying $L_{t+1} = 1.011 L_t$ to L_{1960} (the first observation).
  - Compare the data and the long-run trend.
  - What will be L_t in $t = 2050$?

EXPERIMENTS

FIGURE: U.S. Population, Data and Model's Prediction (Millions)



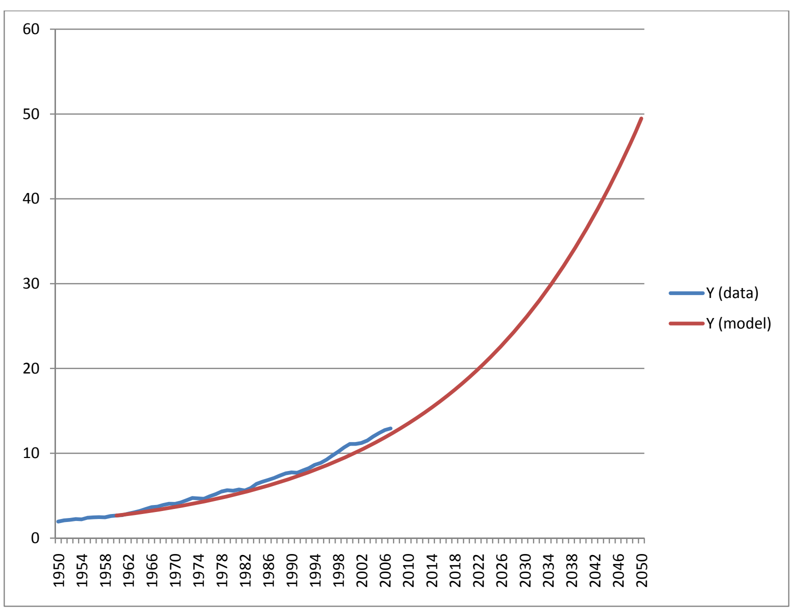
EXPERIMENTS

- Long-run trend of $\{Y_t\}$

- Obtain $\{Y_t\}$ by multiplying POP by rgdpl. (Adjust the units.)
- Obtain the "long-run trend" of $\{Y_t\}$ by applying $Y_{t+1} = 1.033Y_t$ to Y_0.
- Compare the data and the long-run trend.
- What will be Y_t in $t = 2050$?

EXPERIMENTS

FIGURE: U.S. Real GDP, Data and Model's Prediction (2005$Trillions)

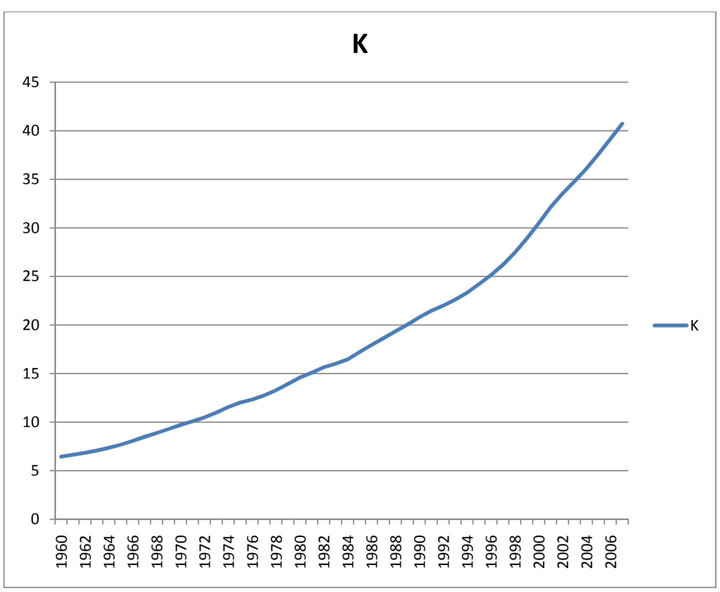


EXPERIMENTS

- Now we are ready to provide annual obs on $\{A_t\}$ and $\{K_t\}$!
- Computing the initial value of K_t in $t = 1960$:
- Since $K/Y = 2.42$, assume $K_{1960} = 2.42Y_{1960} = \$6{,}443$ billion.
- How can we obtain K_{1961}?
- One way is to use $K_{1961} = 2.42Y_{1961}$.
- But this is based on the assumption that $\{s_t\}$ is constant. We do have good observations on $\{s_t\}$ now.
- Use $K_{t+1} = s_t Y_t + (1 - 0.05)K_t$.

EXPERIMENTS

FIGURE: U.S. Physical Capital Stock, Measured by the Model (2005$Trillions)



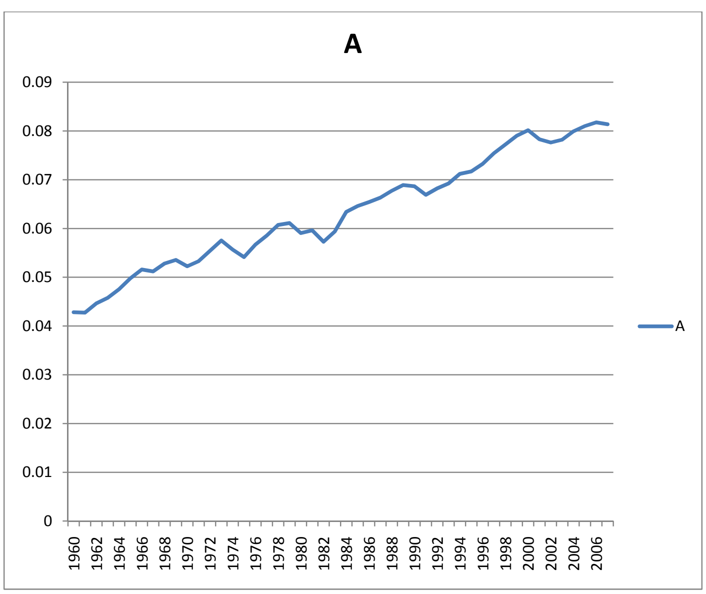
EXPERIMENTS

- What about $\{A_t\}$?

- For each year t: Since $Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}$, measure $A_t = \frac{Y_t}{(K_t)^{0.32}(L_t)^{1-0.32}}$.

EXPERIMENTS

FIGURE: U.S. Total Factor Productivity



EXPERIMENTS

$$Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}. \text{ (Cobb-Douglas)}$$
$$A_{t+1} = 1.015A_t.$$
$$L_{t+1} = 1.011L_t.$$
$$K_{t+1} = s_tY_t + (1 - 0.05)K_t.$$

- We also know on the BGP: $s_t = 0.20$, $g_Y = g_K = 3.3\%$, $K/Y = 2.42$.

- We now measured everything.
- Now we can "predict" the future with the model.
- Of course there is no perfect prediction. We can only provide what the "trend" is likely to be.

EXPERIMENTS

FIGURE: U.S. Future Economic Growth

t	A_t	L_t (million)	K_t (\$trillion)	Y_t (\$trillion)	Y_t/L_t (\$)	s_t
2007	0.081	301.3	40.7	12.9	42,897	0.2
2008						0.2
2009						0.2
2010						0.2
2011						0.2
2012						0.2
…						…

- Now we can fill all the blanks.

EXPERIMENTS

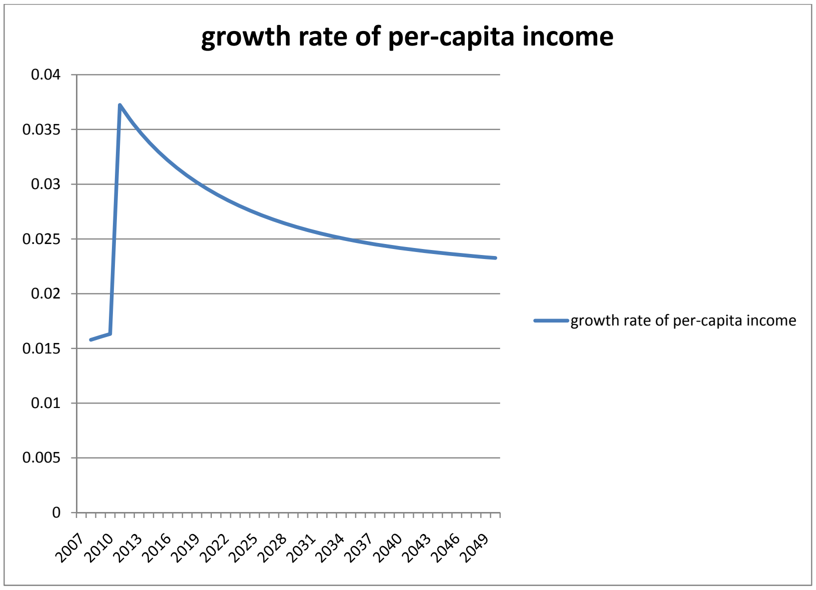
- **Policy Question 1**: What if s_t permanently rises to 0.4 from 2010 on?

FIGURE: U.S. Future Economic Growth

t	A_t	L_t (million)	K_t ($trillion)	Y_t ($trillion)	Y_t/L_t ($)	s_t
2007	0.081	301.3	40.7	12.9	42,897	0.2
2008						0.2
2009						0.2
2010						**0.4**
2011						**0.4**
2012						**0.4**
…						…

EXPERIMENTS

FIGURE: Effect of a Permanent Rise in Investment Rate



EXPERIMENTS

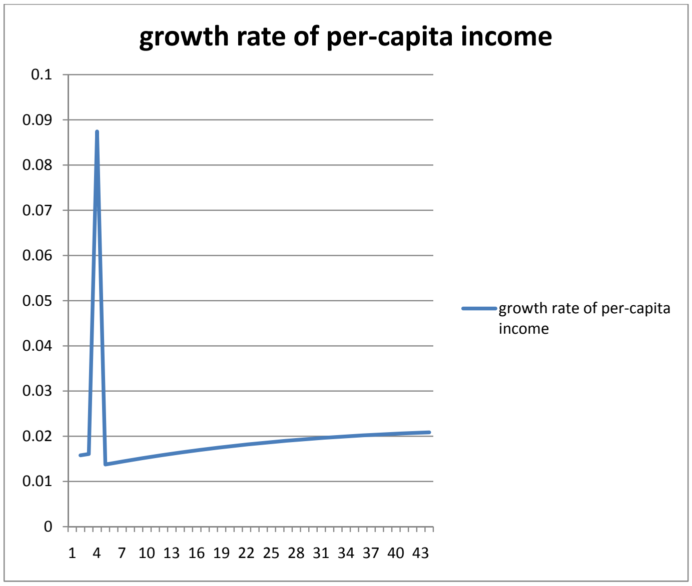
- **Policy Question 2**: What if there is a temporary FDI inflow of $10 trillion in 2010?

FIGURE: U.S. Future Economic Growth

t	A_t	L_t (million)	K_t ($trillion)	Y_t ($trillion)	Y_t/L_t ($)	s_t
2007	0.081	301.3	40.7	12.9	42,897	0.2
2008						0.2
2009						0.2
2010			**ADD 10!**			0.2
2011						0.2
2012						0.2
…						…

EXPERIMENTS

FIGURE: Effect of a Temporary FDI Inflow



EXPERIMENTS

- Permanent rise in s_t
  - A rise in per-capita-GDP growth rate.
  - The effect fades away slowly.
- Temporary FDI inflow
  - A temporary rise in per-capita-GDP growth rate.
  - But the effect is immediately over. Do you want a more sustainable effect? Then you should keep having FDI inflows!
  - We focused only on FDI's effect on K_t. In developing economies, A_t may also be affected.
    (E.g., if Honda builds a factory in China, there will be a technology transfer from Japan to China.)

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GROWTH DECOMPOSITION

- What are the sources of the U.S. growth?

- $Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}$
- $\log(Y_t) = \log(A_t(K_t)^{0.32}(L_t)^{1-0.32})$

$$= \log(A_t) + \log(K_t)^{0.32} + \log(L_t)^{1-0.32}$$

$$= \log(A_t) + 0.32\log(K_t) + (1-0.32)\log(L_t)$$

- $\log(Y_{t+1}) = \log(A_{t+1}) + 0.32\log(K_{t+1}) + (1-0.32)\log(L_{t+1})$
- $\log(Y_{t+1}) - \log(Y_t) = \log(A_{t+1}) - \log(A_t) + 0.32(\log(K_{t+1}) - \log(K_t)) + (1-0.32)(\log(L_{t+1}) - \log(L_t))$
- $g_Y = g_A + 0.32g_K + 0.68g_L$

GROWTH DECOMPOSITION

- Now the growth of real GDP (LHS) is decomposed into (i) a contribution from productivity growth, (ii) a contribution from the growth of physical capital stock, and (iii) a contribution from population growth.
- We know

$$g_Y = g_A + 0.32g_K + 0.68g_L$$

3.3% = 1.5% + 1.1% + 0.7%

(45%) (33%) (21%)

- **Result**: According to this decomposition, 33% of the U.S. economic growth is due to physical capital accumulation. 21% is due to population growth. 45% is basically unexplained.

GROWTH DECOMPOSITION

- **Problem**: g_K is high probably because there is population growth. (That is, a person holds some K. If there is population growth, K will of course increase. The above decomposition does not clearly distinguish g_K from g_L.
- A possible solution: Let's do it with per-capita GDP:

• $Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}$

• $\frac{Y_t}{L_t} = \frac{A_t(K_t)^{0.32}(L_t)^{1-0.32}}{L_t} = A_t(K_t)^{0.32}(L_t)^{-0.32} = A_t\left(\frac{K_t}{L_t}\right)^{0.32}$

• This is the production function in terms of per-capita GDP.

• $\log\left(\frac{Y_t}{L_t}\right) = \log\left(A_t\left(\frac{K_t}{L_t}\right)^{0.32}\right)$

• $\log(Y_t) - \log(L_t) = \log(A_t) + \log\left(\left(\frac{K_t}{L_t}\right)^{0.32}\right)$

GROWTH DECOMPOSITION

• $\log(Y_t) - \log(L_t) = \log(A_t) + 0.32\log\left(\frac{K_t}{L_t}\right)$

• $\log(Y_t) - \log(L_t) = \log(A_t) + 0.32(\log(K_t) - \log(L_t))$

• $\log(Y_{t+1}) - \log(L_{t+1}) = \log(A_{t+1}) + 0.32(\log(K_{t+1}) - \log(L_{t+1}))$

• $g_Y - g_L = g_A + 0.32(g_K - g_L)$

- Now the growth of real per-capita GDP (LHS) is decomposed into (i) a contribution from productivity growth and (ii) a contribution from the growth of per-capita physical capital stock.
- From our calibration,

$$g_Y - g_L = g_A + 0.32(g_K - g_L)$$

2.2% = 1.5% + 0.7%

(68%) (32%)

GROWTH DECOMPOSITION

- Result: According to this decomposition, 32% of the U.S. growth of per-capita GDP is due to physical capital accumulation. 68% is basically unexplained.
- **Problem**: An average person holds more physical capital maybe because productivity is improving over time.
- If A rises → Y rises → K rises (since K/Y is constant) → K/L will of course rise.
- A possible solution: So we don't like K/L. Use K/Y instead. (See Hall and Jones (1999).)
- K/Y is often called a **capital-output ratio**.

GROWTH DECOMPOSITION

- Redo:

- $Y_t = A_t(K_t)^{0.32}(L_t)^{1-0.32}$
- $\frac{Y_t}{(Y_t)^{0.32}} = \frac{A_t(K_t)^{0.32}(L_t)^{1-0.32}}{(Y_t)^{0.32}}$
- $(Y_t)^{1-0.32} = A_t\left(\frac{K_t}{Y_t}\right)^{0.32}(L_t)^{1-0.32}$
- $\left(\frac{Y_t}{L_t}\right)^{1-0.32} = A_t\left(\frac{K_t}{Y_t}\right)^{0.32}$
- $\frac{Y_t}{L_t} = \left[A_t\left(\frac{K_t}{Y_t}\right)^{0.32}\right]^{1/(1-0.32)} = (A_t)^{1/(1-0.32)}\left(\frac{K_t}{Y_t}\right)^{0.32/(1-0.32)}$
- Now the production function is in per-capita units and in terms of K/Y.
- $\log\left(\frac{Y_t}{L_t}\right) = \log\left[(A_t)^{1/(1-0.32)}\left(\frac{K_t}{Y_t}\right)^{0.32/(1-0.32)}\right]$

GROWTH DECOMPOSITION

- $\log\left(\frac{Y_t}{L_t}\right) = \log\left[(A_t)^{1/(1-0.32)}\right] + \log\left[\left(\frac{K_t}{Y_t}\right)^{0.32/(1-0.32)}\right]$
- $\log(Y_t) - \log(L_t) = \frac{1}{1-0.32}\log(A_t) + \frac{0.32}{1-0.32}[\log(K_t) - \log(Y_t)]$
- $g_Y - g_L = \frac{1}{1-0.32}g_A + \frac{0.32}{1-0.32}(g_K - g_Y)$

- From our calibration,

$$g_Y - g_L = \frac{1}{1-0.32}g_A + \frac{0.32}{1-0.32}(g_K - g_Y)$$

2.2% = 2.2% + 0.0%

(100%) (0%)

- Ultimately, something other than physical capital motivates economic growth.

GROWTH DECOMPOSITION

- K is not everything. Need to study other sources.
- Education?
- R&D?
- Knowledge transfer from foreign countries?