EconS 502, Spring 2010
Professor: S. Choi
Due: In Class, Tuesday, 9th February

PROBLEM SET 4

Form a group of 1-4 students. A group can submit one solution.

1. Are the following statements TRUE or FALSE? Provide your theoretical and/or empirical reasonings based on our class discussions. The credits will be solely based on your reasonings.

   (a) Each individual will determine his or her optimal level of schooling. There is no need for the government to intervene in education.

2. Pick up any three country-level economies. For each economy, discuss how human capital accumulation (through education) was quantitatively important in its per-capita output growth. Give a decomposition, for example, as follows:

$$\underbrace{(\text{Per-capita Output Growth})}_{\substack{2.5\% \text{ per year} \\ (100\%)}} = \underbrace{(\text{Contribution from H Accumulation})}_{\substack{1.0\% \text{ per year} \\ (40\%)}} + \underbrace{(\text{Misc.})}_{\substack{1.5\% \text{ per year} \\ (60\%)}}$$

   Measure per-capita human capital stock at time t as $h(t) = \exp(0.1 \times S)$, where S is the average years of schooling for workers of 25 years and above. Use the start year (e.g., 1960) and the end year (e.g., 2000) that are common to your three economies. Determine those years based on data availability.

   - The data on years of schooling can be found at Barro, Robert J. and Jong-Wha Lee (2000), "International Data on Educational Attainment: Updates and Implications," CID Working Paper 42. Visit http://www.cid.harvard.edu/ciddata/ciddata.html and click "Appendix Data Tables".
   - The data on per-capita output growth can be found at many places, but Angus Maddison provides a comprehensive dataset. Click "Statistics on World Population, GDP and Per Capita GDP, 1-2006 AD" at http://www.ggdc.net/maddison. Make sure you use "per-capita GDP" instead of "(total) GDP".

3. References:

Greenwood, J., Z. Hercowitz and P. Krusell (1997), "Long-Run Implications of Investment-Specific Technological Change," American Economic Review, 87(3), 342-362.

Eaton, J., and S. Kortum (2001), "Trade in Capital Goods," European Economic Review, 45(7), 1195-1235.

In class, we discussed Greenwood, Hercowitz and Krusell (1997) to understand the role of a price decline in capital goods. In this problem, you will follow Eaton and Korum (2001) to study the role of importing cheaper capital goods.

Time is dicrete. The world consists of two economies, "North" and "South". "North" produces both types of goods, consumption (C) goods and capital (K) goods. "South" produces C goods only. There is "North-South" trade, in which "North" exports only K goods to "South" and imports C goods from "South". Of course, "South" exports C goods and imports K goods only. Notice that "North" may still produce C goods. That is, C consumption in "North" equals C production in "North" plus C imports from "South".

"North" has L workers. "South" has L^* workers. There is no population growth in either economy. All markets are perfectly competitive. Further descriptions follow:

(i) $L = L_{Ct} + L_{Kt}$: In "North" at period t, out of L workers, L_{Ct} workers work in C industry, and the remaining L_{Kt} workers work in K industry.

(ii) $L^* = L^*_C$: In "South", all L^* workers work in C industry.

(iii) $Q_{Ct} = L_{Ct}^{1-\alpha} K_{Ct}^{\alpha}$: Production function of C goods in "North". Q_{Ct} is output, K_{Ct} is K stock devoted to C production.

(iv) $Q^*_{Ct} = B^* \left(L^*_C\right)^{1-\alpha} \left(K^*_{Ct}\right)^{\alpha}$: "South" counterpart. B^* is the constant productivity.

(v) $Q_{Kt} = A_t L_{Kt}^{1-\alpha} K_{Kt}^{\alpha}$: Production function of K goods in "North". Q_{Kt} is output, A_t is productivity, K_{Kt} is K stock. Notice that α is common in K industry and C industry.

(vi) $A_{t+1} = (1+g)A_t$: Productivity growth in K industry in "North".

(vii) $P^*_C = 1$: The price ($) of C goods in "South" is normalized to 1.

(viii) $d_C > 1$: d_C units of C goods need to be shipped from "South" so that 1 unit of C good arrives in "North". That is, d_C reflects the cost of international trade, including tariffs and transport costs.

(ix) $P_C = d_C P_C^* = d_C$: P_C is the price (\$) of C goods in "North". In equilibrium, C producers in "South" find (i) selling C goods domestically and (ii) exporting them indifferent. By selling 1 unit of C good domestically, a producer receives $P_C^* = 1$. By exporting 1 unit of C good, only $1/d_C$ units arrive in "North", so she receives P_C/d_C. So in equilibrium, $P_C^* = 1 = P_C/d_C$.

(x) $P_{Kt} = P_C/A_t$: In equilibrium, firms in "North" find C industry and K industry indifferent. That is, they all provide zero profit. Now suppose a firm (optimally) hires l units of labor and k units of physical capital.[1] If this firm is in C industry, $l^{1-\alpha}k^{\alpha}$ units are produced and the revenue is $\$P_C l^{1-\alpha}k^{\alpha}$. If this firm is in K industry, $A_t l^{1-\alpha}k^{\alpha}$ units are produced and the revenue is $\$P_{Kt}A_t l^{1-\alpha}k^{\alpha}$. The two revenues are equalized in an equilibrium.

(xi) $P_{Kt}^* = d_K P_{Kt}$: d_K units of K goods need to be shipped from "North" so that 1 unit of K good arrives in "South".

(xii) $K_{t+1}^* = (1-\delta)K_t^* + I_t^*$: "South" law of motion of physical capital, where I_t^* is the units of K goods accumulated.

(xiii) $I_t^* = s^* Y_t^*/P_{Kt}^*$: "South" spends a constant fraction s^* of its income (\$) to buy K goods.

(xiv) $Y_t^* = P_C^* Q_{Ct}^*$: "South" income (\$).

(xv) $K_{t+1} = (1-\delta)K_t + I_t$: "North" law of motion of physical capital, where I_t is the units of K goods accumulated.

(xvi) $I_t = sY_t/P_{Kt}$: "North" spends a constant fraction s of its income (\$) to buy K goods.

(xvii) $Y_t = P_{Ct}Q_{Ct} + P_{Kt}Q_{Kt}$: "North" income (\$).

Answer the following, in a fictional world given in the attached Excel spreadsheet. For simplicity, assume $s = s^*$.

[1] If a firm has \$1, it will spend $\$(1-\alpha)$ to hire workers and $\$\alpha$ to rent the physical capital, in either industry C or K, since either follows Cobb-Douglas with powers $(1-\alpha)$ and α. If wage rate is w and interest rate is r, then $l = (1-\alpha)/w$ and $k = \alpha/r$.

Go as far as you can to answer analytically. Use the "simulation" results, given in the spreadsheet, only as complementary. Provide economic reasonings whenever you can.

(a) Assume a balanced growth path in which all variables grow at constant rates. What is the growth rate of "North" income Y_t, denoted by g_Y? Provide an analytical solution for g_Y, obtain its value with parameter values assumed in rows 3-15 in the spreadsheet, and confirm your result with the simulation.

(b) What about the "South"? Provide an analytical solution for g_{Y^*}, obtain its value with parameter values assumed in rows 3-15 in the spreadsheet, and confirm your result with the simulation.

(c) A traditional (Ricardian) discussion of the "gains from international trade" focuses on the comparative advantage and is based on a "static" gain. However, the model here provides an insight on a "dynamic" gain. Discuss "South"'s gain from the trade with "North". That is, suppose "South" has an initial capital stock of K_0^*. Compare the growth rates of Y_t^* under autarky and under international trade.

(d) d_K includes tariffs. Suppose "South" lowers the tariffs, from the original level of $d_K = 2$ to a new level of $d_K = 1.2$. How are the *long-run* income growth and the income level affected?

(e) Compare P_{Kt}^*/P_C^* and P_{Kt}/P_C. In which economy the relative price of K good to C good higher?

(f) Discuss how this model and its implications are (un)realistic. Suppose that you want to extend this model in your research paper. How will you do it? (You may want to take a quick look at the paper at this point. I don't know whether anyone calibrated this model seriously. If you come up with a good idea, pursue it in your second year. Please feel free to bug me.)