EconS 502: Macroeconomic Theory II
Professor: S. Choi
Due: In Class, Monday, 27th April, 2009

PROBLEM SET 10

Form a group of 1-4 students. A group can submit one solution.

1. In a cash-in-advance model we discussed in class, we considered an economy with stochastic endowments. Here, we consider an economy with (elastic) labor supply where labor productivity is stochastic. To be specific, the represenative consumer has preferences

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\theta}}{1-\theta}-H_t\right)\right],$$

for $0<\beta<1$, $\theta>0$ and $\theta\neq 1$, where C_t is consumption (in units of physical goods) and H_t is labor supply (e.g., hours worked). So this consumer likes more consumption and less hours worked. The production is

$$Y_t = z_t H_t \text{ for all } t = 0, 1, ...,$$

where Y_t is production (in units of physical goods) and z_t is stochastic and exogenously determined labor productivity. This is a closed economy, and there is no storage, so $Y_t = C_t$ for all $t = 0, 1, ...,$ in equilibrium.

In this problem, you will study how money growth (monetary policy) can provide "distortions" to the economy just like taxes (fiscal policy) do. To do this, we first solve the representative consumer's optimization problem without money, and then solve the same problem with cash-in-advance constraint.

   (a) First, suppose there is no money (and disregard the asset market because $Y = C$ anyway). Hence, the consumer can consume its own output (without cash holding). The state variable for the economy, $s = z$, is a first-order Markov process with transition function $F(s'; s)$ for all s. The Bellman equation in this problem is

$$V(s)=\max_{C,H}\left[\frac{C^{1-\theta}}{1-\theta}-H+\beta\int V(s')dF(s';s)\right],$$

subject to $C = zH$. Show that the consumer will choose $H^* = z^{(1-\theta)/\theta}$ and $C^* = z^{1/\theta}$. This is the best that this consumer can obtain, so call it Pareto optimum.

(Hint: In this case, we don't have to worry about envelope condition because only s (exogenous) enters in the value function. Eventually, the problem is as if you are solving $\max_{C,H} \frac{C^{1-\theta}}{1-\theta} - H$ subject to $C = zH$ at each period.)

(b) Now consider a cash-in-advance economy. Similar to the set-up considered in class, the following is a sequence of events at each period t:

1. The productivity shock z_t is realized and the central bank injects (or withdraws) money. Denote by $\omega_t - 1$ the net growth rate of money supply (i.e., $\omega_t = \overline{M}_t/\overline{M}_{t-1}$).

2. The consumer allocates her asset between cash holding, M_t, and the holding of (nominal) bond, N_t, at price Q_{Bt}. To be specific, she pays $Q_{Bt}N_t$ dollars to purchase N_t units of bonds, and at the next period, N_t dollars are delivered.

3. Labor is supplied, production occurs, and consumption is determined. Here, the cash-in-advance constraint, $P_tC_t \leq M_t$, should be satisfied.

Then, the consumer's asset at $t+1$ consists of cash and maturing bonds, plus net receipts from the sale of physical goods at the end of t, plus an injection of cash at the beginning of $t+1$. This implies that the budget constraint at $t+1$ becomes

$$M_{t+1} + \underbrace{Q_{B,t+1}N_{t+1}}_{\text{purchase of bonds}} \leq M_t + N_t + \underbrace{P_t(\underbrace{z_tH_t}_{=Y_t} - C_t)}_{\text{net sale of goods}} + \underbrace{(\omega_{t+1} - 1)\overline{M}_t}_{\text{injection of cash}}.$$

The state variable for the economy, $s = (z, w)$, is a first-order Markov process with transition function $F(s'; s)$ for all s. We look at equilibria in which $P_t = \overline{M}_t p(s_t)$ and $Q_{Bt} = q_b(s_t)$. Define $m_t = \frac{M_t}{\overline{M}_t}$ and $n_t = \frac{N_t}{\overline{M}_t}$. The Bellman equation is then

$$V(\alpha, s) = \max_{C,H,m,n} \left[\frac{C^{1-\theta}}{1-\theta} - H + \beta \int V(\alpha', s') dF(s'; s) \right]$$

s.t.

$$p(s)C \leq m,$$
$$m + q_b(s)n \leq \alpha/\omega$$
$$\alpha' = m + n + p(s)(zH - C) + \omega' - 1$$

Write down the first-order conditions and envelope condition.

(c) We look for an equilibrim in which quantities (such as C and H) and normalized prices (such as p and q_b) are functions only of the current state. Equilibrium conditions are $m = 1$, $n = 0$, and $C = zH$. We conjecture that in equilibrium, no excess cash is held, so the cash-in-advance condition holds with equality: $C = 1/p(s)$. Rearranging your first-order conditions and envelope condition, show that in equilibrium,

$$H(s) = \beta E\left[\left.\frac{(z'H(s'))^{1-\theta}}{\omega'}\right| s\right] \text{ (determination of } H\text{'s)}$$

$$q_b(s) = \beta E\left[\left.\frac{1}{\omega'}\left(\frac{z'H(s')}{zH(s)}\right)^{1-\theta}\right| s\right] \text{ (determination of bond price)}$$

$$p(s) = \frac{1}{zH(s)} \text{ (determination of good price)}$$

(Notice that the determination of H is different from the one you obtained in (a). Determination of H depends on future money growth, ω'. By introducing money, the economy is distorted!)