

EconS 502: Macroeconomic Theory II
 Professor: S. Choi
 Due: In Class, Monday, 9th February

PROBLEM SET 3

1. Consider the following economy. A representative consumer solves

$$\max_{\{C(t)\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt, \quad \sigma > 0, \quad \sigma \neq 1,$$

where $C(t)$ is aggregate consumption. There is no population growth or productivity growth. The only source of growth is physical capital accumulation. So the constraints are

$$Y(t) = B(1 - \tau)K(t), \quad 0 \leq \tau < 1, \quad B > 0,$$

$$Y(t) = C(t) + I(t),$$

$$\dot{K}(t) = I(t) - \delta K(t), \quad 0 < \delta < 1,$$

where $Y(t)$ is aggregate output, B is constant productivity, τ is a flat tax rate on physical capital, $K(t)$ is the stock of physical capital, $I(t)$ is aggregate investment, and δ is a depreciation rate.

- (a) Set up the Hamiltonian function and obtain first-order conditions.
- (b) Assume a balanced growth path in which all variables grow at constant rates. Calibrate the model using the following observations. Assume $\sigma = 2$.
 - $\tau = 36\%$: McGrattan and Prescott (Federal Reserve Bank of Minneapolis Quarterly Review, 2000), Table 2.
 - $s \equiv I_t/Y_t = 0.20$: 1960-2005, NIPA Table 1.1.5.
 - $g_Y = 2.2\%$: 1960-2005, NIPA Table 7.1. This is the growth of GDP per worker. Yes, $Y(t)$ is aggregate GDP, not GDP per worker. But when the model doesn't have population growth, it is more reasonable to use the growth of GDP per worker because we get the same result in that way as in a model with population growth.
 - $\delta K/Y = 0.12$: 1960-2005, NIPA Table 1.1.
- (c) If taxes on physical capital income are eliminated so that $\tau = 0$, what would be the new growth rate of $Y(t)$ on the new balanced growth path?
- (d) What would be the new level of s on this new balanced growth path?