Cross-Sectional Returns in a General Equilibrium: Is There a Momentum?

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Midwest Finance Association
February 2010

Introduction

2 major contributions:

- *Theoretical/Empirical*: Derived **quantitative** implications on
  1. an average **individual stock**.
  2. the **momentum** and other **cross-sectional properties**.
- *Technical*: Solved an **orchard** model with the **recursive utility**.

1. Theoretical/Empirical Contribution

- **Momentum Effect:** Buying last year's top 10% winners and short-selling the bottom 10% losers $\Longrightarrow$ 8.2% averege annual profit. (Jegadeesh and Titman, 1993)
- Why is this puzzling?
  1. Under **Efficient Market Hypothesis (EMH)**,
     - Can't make above-normal profits with currently available information.
     - Fama and French's (1996) 3-factor model does not seem to explain it. EMH wrong?
  2. In **behavioral finance**,
     - "Investors underreact to firm-specific news, compared to market-level news."
     - But the momentum is strong for well-diversified portfolios and then vanishes at the market level? (Lewellen, 2002)

1. Theoretical/Empirical Contribution

- **This Paper**: Under EMH,
  - If a stock's realized dividend growth is higher, then the dividend is now responsible for a higher share of the consumption.
  - The change can make the wealth portfolio **less diversified**, discouraging the **risk-averse representative investor** to hold it.
  - But a **market equilibrium** makes her happy to do so, by decreasing a market price of the stock.
  - The stock now has a **higher ER**, as a compensation for a less diversified wealth portfolio.
- **Literature**:
  - Cochrane, Longstaff and Santa-Clara (2008, RFS): Quantitatively small to justify the momentum with log utility.

2. Technical Contribution

- **Orchard Model**:
  - Cochrane, Longstaff and Santa-Clara (2008, RFS): **Log utility**, 2 trees.
    - Faces **equity premium** puzzle
    - Faces **risk-free rate** puzzle
  - Martin (2009, unpublished): **CRRA**, 2 trees.
  - This paper: **Recursive utility**, 2 trees.
  - Also related:
    - Menzly, Santos and Veronesi (2004): CRRA+Habit, multiple trees, but with complicated dividend growth

1. The Orchard

- **Endowment Economy**: $C_t = D_t^A + D_t^B$
- Two trees, A and B, provide D_t^A and D_t^B units as dividends.
- Div. growths are i.i.d. over periods.
- **State Variable**: $s_t \equiv \frac{D_t^A}{D_t^A + D_t^B}$

	Prob.	A's dividend growth, D_{t+1}^A / D_t^A	B's dividend growth, D_{t+1}^B / D_t^B	Cons. growth, C_{t+1} / C_t
#1	π_1	g_H^A (High)	g_H^B (High)	$g_H^A s_t + g_H^B (1 - s_t)$
#2	π_2	g_H^A (High)	g_L^B (Low)	$g_H^A s_t + g_L^B (1 - s_t)$
#3	π_3	g_L^A (Low)	g_H^B (High)	$g_L^A s_t + g_H^B (1 - s_t)$
#4	π_4	g_L^A (Low)	g_L^B (Low)	$g_L^A s_t + g_L^B (1 - s_t)$

2. Euler Equation

- **Preferences** of the **Representative Investor**:

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta(R_t(V_{t+1}))^{1-\rho}\right]^{\frac{1}{1-\rho}} \quad (1)$$
$$R_t(V_{t+1}) = [E_t V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}.$$

  - ρ: an inverse of intertemporal elasticity of substitution
  - γ: risk aversion parameter. If $\rho = \gamma$, the preferences collapse to CRRA with parameter γ.

- **Euler equation** for one-period asset j as

$$P_t^j = E_t\left[m_{t+1} X_{t+1}^j\right], \quad (2)$$
$$m_{t+1} = \beta\left(\frac{V_{t+1}}{R_t(V_{t+1})}\right)^{\rho-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}.$$

  - P_t^j: asset j's price at period t
  - X_{t+1}^j: asset j's random payoff at period $t+1$

3. Solving for Individual Returns

- Rearranging: Divide (1) by C_t,

$$\frac{V_t}{C_t} = \left[(1-\beta) + \beta \left[E_t \left(\frac{C_{t+1}}{C_t} \frac{V_{t+1}}{C_{t+1}} \right)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right]^{\frac{1}{1-\rho}} .$$

- Define $V_t^* \equiv V_t / C_t$.
- (2) applied for tree A: $P_t^A = E_t \left[m_{t+1} (D_{t+1}^A + P_{t+1}^A) \right]$.
- Divide by C_t and define $P_t^{A*} \equiv P_t^A / C_t$ to be **P/C ratio**.

$$P_t^{A*} = E_t \left[\beta \left(\frac{V_{t+1}^*}{\left[E_t \left(\frac{C_{t+1}}{C_t} V_{t+1}^* \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left(s_{t+1} + P_{t+1}^{A*} \right) \right] .$$

3. Solving for Individual Returns

- s_t is the only state variable in this economy.
- **Problem**: Solve for $V^*(s)$ and $P^{A*}(s)$.
- Can be done numerically.
- **P/D ratio** is obtained as

$$\frac{P_t^A}{D_t^A} = \frac{P_t^{A*}}{s_t}.$$

- The **(net) return on tree A** at period $t+1$, denoted by r_{t+1}^A, is

$$1 + r_{t+1}^A \equiv \frac{D_{t+1}^A + P_{t+1}^A}{P_t^A} = \frac{C_{t+1}}{C_t} \frac{s_{t+1} + P_{t+1}^{A*}}{P_t^{A*}}.$$

1. Summary

Matching Data to the **Orchard**

- Set-up: 5,000 identical stocks. A=(1 stock). B=(4,999 stocks). $s_t = 0.02\%$.
- $g_H^B = 1.06$ and $g_L^B = 0.98$, with $\pi_1 + \pi_3 = \pi_2 + \pi_4 = 1/2$ (Mehra and Prescott, 1985)
- $g_H^A = 1.12$, $g_L^A = 0.92$, $\pi_1 = \pi_4 = 0.35$, and $\pi_2 = \pi_3 = 0.15$ (selected to match $E[r_t^A|s_t = 0.02\%] = E[r_t^B|s_t = 0.02\%]$ and a "restriction" from "identical stocks" assumption.)
- $\beta = 0.97$ (conventional)
- $\gamma = 57.5$, $\rho = 0.6$ (Chen, Favilukis and Ludvigson, 2009)

1. Summary

Values Calibrated from the Model

- $P^{A*}(s_t = 0.02\%)/0.02\% = 30.0$: **Individual P/D**
- $P^{B*}(s_t = 0.02\%)/99.98\% = 29.2$: **Market P/D** (Data: 30, Menzly, Santos and Veronesi, 2004)
- $E[r^A_{t+1}|s_t = 0.02\%] = 5.4\%$: **Individual ER**
- $E[r^B_{t+1}|s_t = 0.02\%] = 5.5\%$: **Market ER** (Data: 7.7%)
- $r^f(s_t = 0.02\%) = 1.5\%$: **Risk-free rate** (Data: 0.6%)

1. Summary

Values Calibrated from the Model (cont'd)

- (slope)= 0.08: Regression slope of **expected excess returns** on **CAPM betas** (Data: 0.08, Cochrane, 2005, Chap. 20)
- $\sigma[r^A_{t+1}|s_t = 0.02\%] = 10.1\%$: **Individual return volatility** (Data: 49%, Statman, 1987)
- $\sigma[r^B_{t+1}|s_t = 0.02\%] = 4.1\%$: **Market return volatility** (Data: 18%, Statman, 1987)
- $\frac{\sigma[r^B_{t+1}|s_t=0.02\%]}{\sigma[r^A_{t+1}|s_t=0.02\%]} = 0.4$: **Relative volatility** (Data: 0.4, Statman, 1987)

2. Predictive Power of P/D Ratio

- Empirical Literature: Menzly, Santos and Veronesi (2004) report a slope of **2.9**, in a pooled regression of industry-level returns on lagged dividend yields.
- Now assume **5 unidentical portfolios** with equal weights.
- Our model has two trees only. Solve the model 5 times for each portfolio to approximate the solution.
- **Set-Up**: The economy is "good" or "bad" with probability 1/2, i.i.d. over periods. If "good," each portfolio has
  - "high" div. growth with prob $1/2 + \varepsilon$
  - "low" div. growth with prob $1/2 - \varepsilon$, and vice versa.
- Try two cases, $\varepsilon = 0.2$ and $\varepsilon = 0.4$. 5 potfolios have expected div growths of 1, 1.01, ..., 1.05. Volatilities are determined to match the consumption data.

2. Predictive Power of P/D Ratio

(A) $\varepsilon = 0.2$

Portfolio	g_H^j	g_L^j	Div Yield (1/(P/D))	ER
#1	1.02	0.98	2.8%	3.2%
#2	1.05	0.97	3.4%	4.6%
#3	1.08	0.96	3.9%	5.9%
#4	1.11	0.95	4.5%	7.4%
#5	1.14	0.94	5.2%	8.9%
			Slope:	2.4

(B) $\varepsilon = 0.4$

...

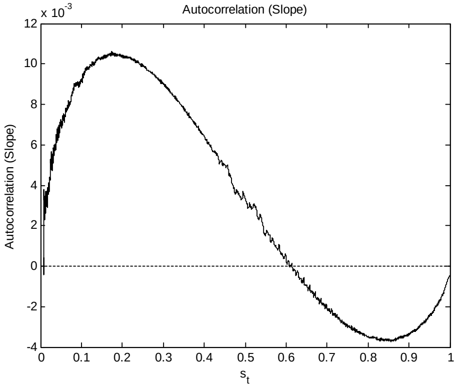
2. Predictive Power of P/D Ratio

- Our "laboratory" is able to generate reasonable features of the predictability.
- Is the lab an environment that is close to the reality?
- Possible Approaches:
  - Further investigate whether it can also provide other reasonable predictions. (e.g., momentum)
  - Try values for g_H^j and g_L^j so that portfolio returns match reasonably with the observed on size-sorted, book-to-market-sorted, and industry-level portfolios. (Future work)

Momentum

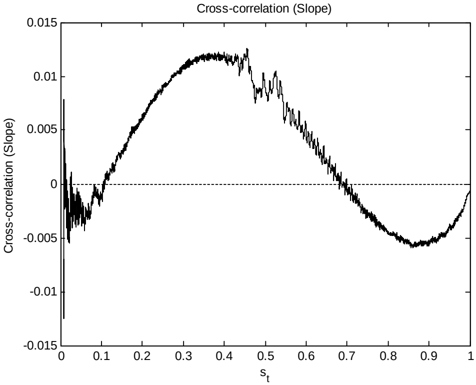
- Lewellen (2002) decomposes the momentum profit into three sources:
  - **(i)** a **positive autocorrelation** of stock (or portfolio) j's returns
  - **(ii)** a **negative cross-serial correlation** between stock j's and stock i's returns ($j \neq i$),
  - (iii) the levels of unconditional expected returns.
- Plan:
  1. A=(average individual stock). Look for two sources, (i) and (ii). Check the signs.
  2. The same lab of 5-portfolio economy. Simulate the way Lewellen (2002) computes the momentum from the data.

- **Source (i)**: We obtain $cov_t(r^A_{t+1}, r^A_{t+2})/var_t(r^A_{t+1})$.



Momentum

- **Source (ii)**: The regression slopes of tree A's returns in $t+2$ on tree B's return in $t+1$, i.e., $cov_t(r^B_{t+1}, r^A_{t+2}) / var_t(r^B_{t+1})$.



Momentum

- **Source (i)**
  - When $s_t \longrightarrow 0$ or 1, the autocorrelation $\longrightarrow 0$ because the model collapses to a one-tree model and does not provide any diversification motives.
  - **Autocorrelation** is **positive** when $s_t = 0.02\%$.
- **Source (ii)**
  - **Cross-serial correlation** is **negative** when $s_t = 0.02\%$.
- Consistent with Lewellen's (2002) empirical result.
- But what about *numbers*, not *signs*?

Momentum

- In Lewellen (2002), portfolio j's weight in year $t+1$ is formed by $w_{t+1}^{j} = (1/N)(r_t^j - r_t^W)$.
  - N: the total number of portfolios (between 5 and 25).
  - r_t^W : period-t return on wealth portfolio.
- Consider a momentum strategy, investing \$1 long and \$1 short, respectively.
- In 1963-1999, this strategy generates an average profit **between \$0.035 and \$0.056**.
- Simulate in our laboratory!

Momentum

(A) $\varepsilon = 0.2$

Prob.	Agg. Shock	Div. Growth	r^j_{t+1}	$E[r^j_{t+2} \| r^j_{t+1}, s_t = 0.2]$
8.4%	"Good"	P#1: H	$= 5.6\%$	$= 3.2\%$
		P#2: H	$= 8.8\%$	$= 4.6\%$
		P#3: H	$= 11.8\%$	$= 5.9\%$
		P#4: H	$= 15.1\%$	$= 7.4\%$
		P#5: H	$= 18.1\%$	$= 8.8\%$
...	...	...	...	...

- Set up the momentum portfolio: Long (P#4: \$0.34, P#5: \$0.66) Short (P#1: \$0.67, P#2: \$0.32, P#3: \$0.01.)
- Expected profit in this scenario: \$0.05

Momentum

- The **momentum profit**: \$0.010 for $\varepsilon = 0.2$, \$0.059 for $\varepsilon = 0.4$.
- Hence, $\varepsilon = 0.4$ is close to what Lewellen (2002) reports.
- Even $\varepsilon = 0.2$ is not discouraging.
  - The momentum arises because returns are volatile, having different values across the states.
  - Our return volatility is low compared to the data.

Conclusion

- What this paper **did** to understand the momentum
  - Autocorrelation is $(+)$.
  - Cross-serial correlation between period-t return on tree A and period-$(t+1)$ return on tree B is $(-)$.
  - Approximated a 5-portfolio world. The momentum profit is substantial.
- What this paper **didn't do**:
  - Return volatility is not enough to match the data.
  - A general N-tree model is not solved.
- Conclusion:
  - General equilibrium is potentially a key to understand the momentum.
  - If all risk factors are *completely* controlled for in an empirical analysis, most of the momentum could disappear.